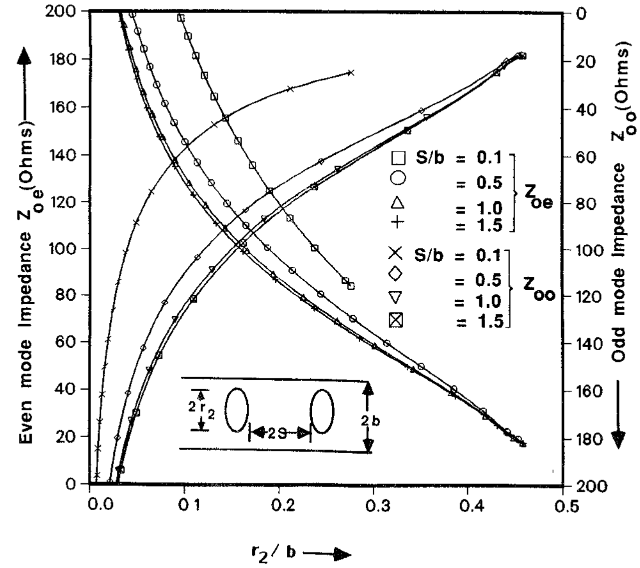

Fig 5. Variation of even- and odd-mode impedances as a function of r_2/b for $k = 0.5$.

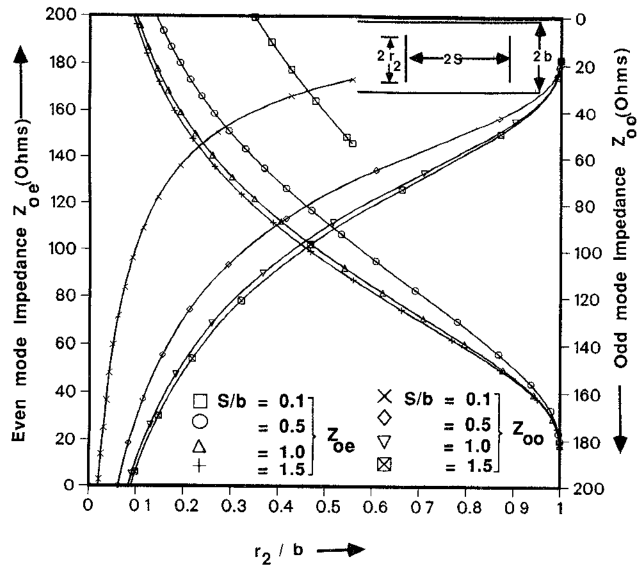

Fig. 6. Variation of even- and odd-mode impedances as a function of r_2/b for $k = 0\ 0$

IV. Conclusions

Data on a generalized structure of elliptical (oval) conductors have been presented from the available analysis of some of the basic transmission structures. With a single set of equations, different shapes of the center conductors of the coupling structures can be analyzed by choosing the parameter α. Close agreement of the results of the present formulation with some special cases available in the literature confirms the validity of the analysis. This formulation can also be used to find the impedance of a transmission structure having a center conductor of finite thickness with rounded corners by choosing a low value of the compression ratio. Such structures are useful for high-power applications.

TABLE II
Comparison between the Even- and Odd-Mode Impedances Obtained from Levy's Method and the Present Work for the Case of Coupled Circular Bars

		Z_{oe}(ohm)			Z_{oo}(ohm)		
r/b	S/b	Levy	present work	%error	Levy	present work	%error
.354	.176	96.2867	95.6461	0.6653	49.9141	48.2222	3.3896
.400	.200	84.8239	84.1968	0.7393	47.9315	46.4332	3.1259
.400	.226	83.6271	82.9573	0.7985	49.9962	48.6779	2.6368
.436	.280	74.9187	74.2094	0.9468	49.9320	48.9281	2.0105
.462	.338	68.8971	68.2559	0.9307	50.0181	49.2143	1.6070
.482	.398	64.5025	63.8566	1.0014	50.0393	49.3000	1.4774
.400	.400	78.1118	77.5973	0.6587	58.9841	58.4546	0.8892
.498	.462	61.0799	60.2663	1.3320	49.9750	49.1582	1.6344
.510	.528	58.5572	57.7676	1.3484	49.9233	49.1648	1.5193
.400	.600	74.1831	73.7622	0.5674	64.0618	63.6594	0.6281

References

[1] S. B. Cohn, "Shielded coupled-strip transmission line," *IRE Trans Microwave Theory Tech.*, vol. MTT-3, pp. 29–38, Oct. 1955.

[2] H. A. Wheeler, "The transmission line properties of a round wire between parallel planes," Wheeler Monogr. 19, June 1954.

[3] Edward G Cristal, "Coupled circular cylindrical rods between parallel ground planes," *IEEE Trans. Microwave Theory Tech.*, vol MTT-12, pp 428–439, July 1964.

[4] E G. Cristal, "Data for partially decoupled round rods between parallel ground planes," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 311–314, May 1968.

[5] R. M Chisholm, "The characteristic impedance of trough and slab lines," *IRE Trans. Microwave Theory Tech.*, vol MTT-4, pp. 166–172, July 1956

[6] Ralph Levy, "Conformal transformations combined with numerical techniques, with applications to coupled-bar problems," *IEEE Trans Microwave Theory Tech.*, vol. MTT-28, pp. 369–375, Apr 1980.

[7] K. V. S. Rao and B. N. Das, "Stripline using an oval shaped center conductor between ground planes," *Proc.*, *Inst. Elec. Eng.*, pt H, pp. 366–368, Dec. 1982.

[8] B. N Das, K V Seshagiri Rao, and A. K Mallick, "Analysis of an oval symmetrically located inside a rectangular boundry by conformal mapping," *IEEE Trans. Microwave Theory Tech.*, vol MTT-31, pp. 403–406, May 1983.

[9] K. V. Seshagiri Rao, B. N. Das, and A K. Mallick, "Characteristic impedance of an oval located symmetrically between the ground planes of finite width," *IEEE Trans Microwave Theory Tech.*, vol MTT-31, pp. 687–681, Aug. 1983.

Numerical Conformal Transformation of Three-Magnetic-Wall Structures

EUGENIO COSTAMAGNA

Abstract **—The traditional conformal transformation approach for capacitance and impedance evaluation in TEM transmission lines leads to the mapping of the line structure into a rectangular geometry.**

Manuscript received October 5, 1988; revised March 16, 1989 This work was supported in part by Marconi Italiana.

The author is with the Istituto di Elettrotecnica, University of Cagliari, Piazza d'Armi, 09123 Cagliari, Italy.

IEEE Log Number 8928331

This approach is not suitable to cope with general two-conductor, three-magnetic-wall line structures, and this paper describes how conformal transformations may transform these structures into less complicated geometries, which either can be solved or have been solved previously by numerical techniques.

I. Introduction

Notable interest has recently been devoted, in TEM and quasi-TEM transmission line analysis and design, to relatively nontraditional conformal mapping approaches, i.e., to combine the advantages of both analytical conformal transformations and numerical or variational methods [1], [2], to enhance the capabilities of purely numerical procedures for the inversion of the Schwarz–Christoffel (SC) formula [3], and to apply these procedures to various structures [4].

In a similar vein, the analysis of general two-conductor, three-magnetic-wall structures by inversion of the SC formula is considered in this paper. It can be noted that a particular case, with two magnetic walls placed at infinity, has been analyzed in [5].

In the traditional approach to the capacitance calculation, conformal transformations are applied aiming at a rectangular configuration, with two parallel electrode sides and two parallel magnetic wall sides, in which the capacitance can be immediately evaluated. This approach cannot be applied to the case of three-magnetic-wall geometries, and two solutions are proposed for the problem in an attempt to preserve similar simplicity in the capacitance evaluation of the transformed geometry.

The first solution is the conformal transformation of the original structure to a new, three-magnetic-wall geometry, for which accurate variational solutions have already been provided, i.e., the even-mode strip geometry as considered in [6] (with an electric instead of magnetic top wall) and [7]. The second is the conformal transformation of the original structure to a suitable four-magnetic-wall structure, the shape of which will allow an immediate parallel-plate capacitance calculation, as soon as two of the magnetic walls have been led to coincide.

Both procedures can be divided into three steps. First, the polygonal boundary of the original structure in the complex w plane is mapped into the real axis of an intermediate z plane by means of inversion of the SC formula. Second, a direct SC transformation into a new w' plane is performed, taking care of suitable constraints imposed on the final w'-plane geometry by assuming suitable vertices and by performing a suitable side length optimization in the z plane. Third, the capacitance evaluation for this geometry is carried out.

II. Conformal Transformation to Even-Mode Stripline Structures

Starting from a general three-magnetic-wall structure in the w plane, as in Fig. 1(a), a closed geometry as shown in Fig. 1(c) can be obtained in the final w' plane by means of a suitable choice of the vertex exponents μ_i in the SC formula [8] during the direct mapping from the intermediate z plane. In the figures, continuous lines denote electric walls, dashed lines magnetic walls.

It is evident that a particular case of this geometry can be selected by imposing equal lengths for the w' plane sides CD and DE, thereby obtaining the even-mode strip geometry in Fig. 1(d). For this purpose, in place of the old point D (if any), a new point D' between C and E has to be assumed as a $\mu = -1$ vertex during the conformal transformation from the z plane to the w' plane, and its z plane position has to be determined merely by imposing equal lengths on the CD' and $D'E$ w' plane sides. This can easily be obtained by means of a suitable optimization

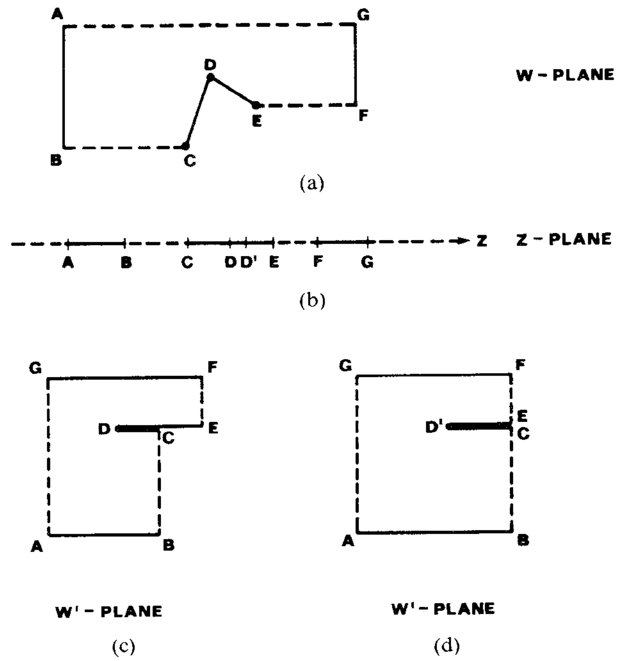

Fig. 1. Conformal transformation of a general three-magnetic-wall structure to an even-mode strip structure (a) General structure in the original w plane (b) Intermediate z plane. (c) Transformed geometry derived from the z plane positions of the w plane vertices. (d) Stripline structure obtained by optimization of the z plane position of the point D'.

procedure applied to the w' plane side lengths, as computed by the SC formula from the actual D' position in the z plane. It can be noted that an analytical solution to the problem of imposing equal lengths on the CD and DE w' plane sides has been given in [9] in a different application for a geometry similar to that of Fig. 1(c), in which the position of the point D in the z plane is optimized, provided that certain constraints on the positions of the C and E points are satisfied.

As a first example of the capabilities of the method, the even-mode angular offset stripline geometry of Fig. 2 is considered: α is the offset or twist angle of the strip with respect to the customary "parallel" geometry.

The corresponding odd-mode structure (electric sidewalls) has been studied in [4] by mapping into a rectangle a geometry derived from it, in which two polygonal magnetic walls have been imposed along the flux lines leaving the top and bottom strip centers. The shape of the flux lines has been determined by analyzing simpler structures, and the sensitivity of the capacitance to their geometry has been found to be very low [4].

Similar techniques have been applied to the BC and EF magnetic walls in Fig. 2, and the whole even-mode three-magnetic-wall geometry has been mapped to the stripline geometry in Fig. 1(d) by optimizing the position of the z plane $\mu = -1$ vertex by means of standard Newton–Raphson or bisection techniques.

Both Gish and Graham's [6] and Smith's [7] procedures have been utilized for the impedance evaluation on the stripline geometry of Fig. 1(d), and the agreement between the results has proved to be very good, normally of the order of some 0.1–0.2 percent. The impedance results are shown in Fig. 2, with dots representing computed values.

Of course, variational calculations can immediately be carried out in the $\alpha = 0°$ cases, and no optimization of the D' point position is necessary in the $\alpha = 90°$ cases. In some large α cases, reference has been made to simplified geometries, due to difficulties in the numerical inversion of the SC formula for geometries

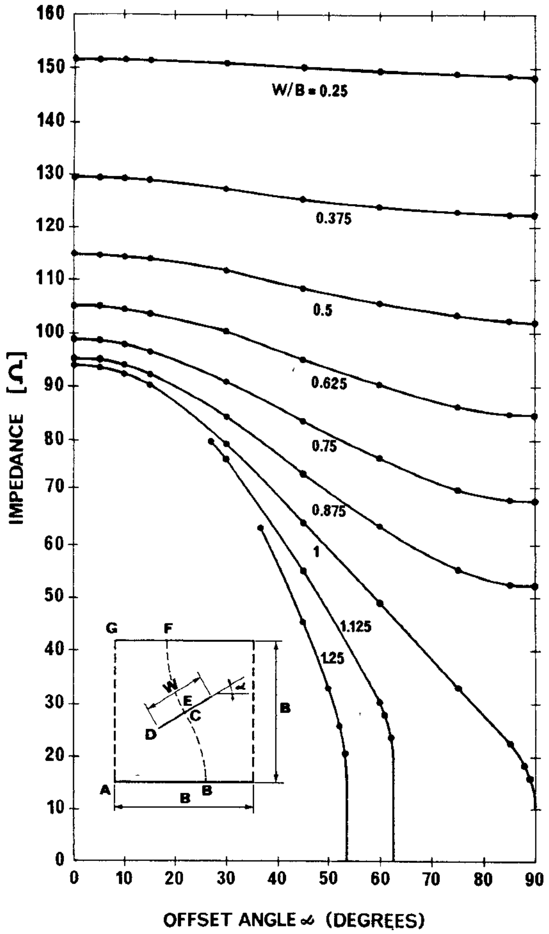

Fig. 2. Characteristic impedance curves for an even-mode angular offset stripline ($\epsilon_r = 1$).

showing very short gaps between the strip edge and the outer conductor, or due to uncertainties in assuming correct geometries for the BC and EF flux lines.

The inversion of the SC formula has been performed by the procedures described in [3]. Partition techniques for the integration intervals have been necessary in many cases, and the computing (CPU) times for the overall impedance calculation on a Digital VAX 8600 computer ranged from some tens of seconds for the simplest cases to several minutes for difficult cases.

As a second example of the capabilities of the method, calculations performed on the slot-coupled trough lines discussed in [2] can be presented. The structure is shown in Fig. 3(a) and the geometry utilized for the present calculations is shown in Fig. 3(b). The aim was to make a comparison between the design curves derived by the present techniques, which are shown in Fig. 4 with thick lines, and the curves already obtained in [2] (thin lines). The abscissa is the even-mode impedance Z_{0e} of the structure in Fig. 3(a); Z_{0e} and the odd-mode impedance Z_{0o} have to be matched according to the equation $\sqrt{Z_{0e} \cdot Z_{0o}} = 50\ \Omega$ [2] (a permittivity $\epsilon_r = 2.17$ has been assumed, as in [2]).

Values for the W and h parameters in Fig. 3(a) have been derived in the following way. For any width W, first the odd-mode impedance Z_{0o} has been calculated on the geometry of Fig. 3(b) by assuming an electric wall on the IL side and by mapping the structure into a rectangle, following the customary procedure for two-magnetic-wall structures. Magnetic walls have been assumed on the BC and GH sides, and the sensitivity of the impedance values to their position has proved to be very small.

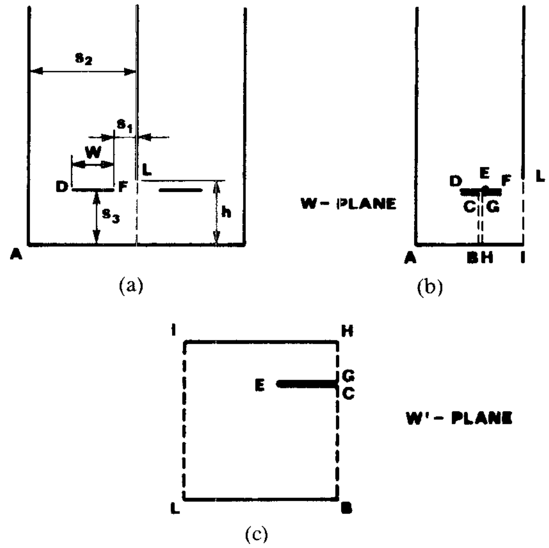

Fig. 3. (a) A pair of slot-coupled trough lines from [2] (b) Three-magnetic-wall geometry derived from it. (c) Even-mode strip structure obtained by conformal transformation

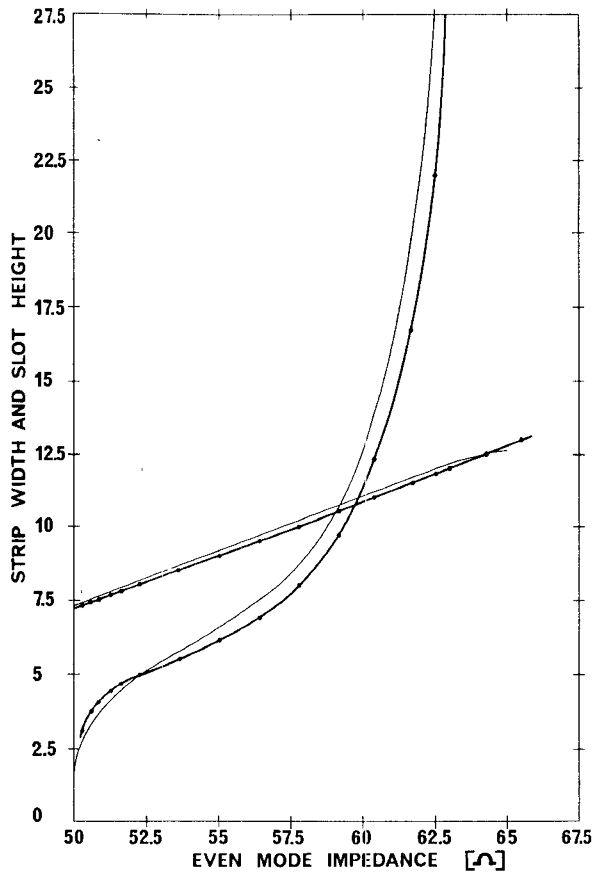

Fig 4. Design curves for the pair of slot-coupled trough lines in Fig. 3 ($S_1 = 1$, $S_2 = 36$, $S_3 = 5$, $\epsilon_r = 2.17$) Thick lines: present theory; thin lines: curves in [2].

From the obtained Z_{0o} and the matching equation, the required Z_{0e} value has been computed, and then the required height h has been derived by means of tentative calculations from assumed trial values and linear interpolation.

For any h value, a numerical inversion of the SC formula has been performed for the polygon $ABCDEFGHIL$, having assumed a magnetic wall on the IL side, and the polygon has been mapped from the intermediate z plane to the even-mode stripline geometry in Fig. 3(c) by means of Newton–Raphson optimization of the z plane position of the point E, mapped into the strip edge point. Finally, impedance calculations have been performed following [6] and [7] with near coincident results (except for a very few points, in which differences of the order of 1 percent have been observed).

The computed W and h pairs are marked by dots in Fig. 4. As shown, the curves obtained following the present three-magnetic-wall approach are very similar to those derived by the discrete variational conformal (DVC) approach in [2]. Moreover, reading W and h pairs on the curves of [2] and computing even- and odd-mode impedances following the present approach, the matching equation is satisfied to a very good approximation, with errors of the order of 1 percent or of some 0.1 percent. This confirms the interest for design purposes of both techniques, and of similar mixed technique approaches.

III. Conformal Transformation to a Four-Magnetic-Wall Structure

A second way to analyze three-magnetic-wall structures is the conformal transformation from the w plane of Fig. 5(a) to the w' plane geometry of Fig. 5(c). This is an improvement of a procedure introduced by Maltese [10] to perform approximate impedance calculations of even-mode impedances for stripline structures, which were transformed in "quasi-parallel-plate" geometries, similar to Fig. 5(c) but with EF and GH side lengths respectively different from CB and AH.

The same parallel-plate geometry of Fig. 5(c) can be obtained by numerical optimization of the z axis position of the point H in Fig. 5(a). The capabilities of the method can be illustrated by considering the same structures analyzed in the previous paragraph. In any case, the impedance values computed in this way have been found to be almost exactly equal to those computed by transforming the structure in a stripline geometry.

Compared to the mapping to an even-mode stripline geometry, the mapping to a parallel-plate geometry exhibits the merit of being mathematically exact. However, it often leads to w plane geometries which are more complex or more critical with respect to the ratios between the lengths of contiguous sides in the z plane, at least when utilizing the simple optimization algorithm described in [3], which can be heavily affected by the choice of the w plane side mapped at infinity in the z plane.

In fact, calculation of the impedance for some points in Fig. 2 has been made possible only by introducing some improvements in the procedure, i.e., by performing first the inversion of a structure like the polygon $ABCDEFG$ in Fig. 5(a), and by introducing at a later time the sides PA and GQ in the intermediate z plane of Fig. 5(b) before the final mapping to the w' plane. Care has been taken to select z plane lengths sufficient to give a negligible PQ length in the original w plane, but not so large as to give rise to length ratio problems in the z plane. Then, the z plane length ratio between the PA and GQ sides has been optimized to obtain equal lengths for the GQ and PA sides in the w' plane geometry in Fig. 5(d), in which the PQ side is parallel to EC, and finally the length of the PQ side is checked to have really assumed a negligible value.

IV. Conclusions

Procedures for the analysis of three-magnetic-wall structures by means of conformal mapping techniques have been introduced, and some examples have been presented. The accuracy of the results seems to be suitable to emphasize in general the capabilities of mixed conformal mapping and variational techniques, as well as the usefulness of optimization procedures performed on specific parameters of the final geometry obtained by the mapping procedure.

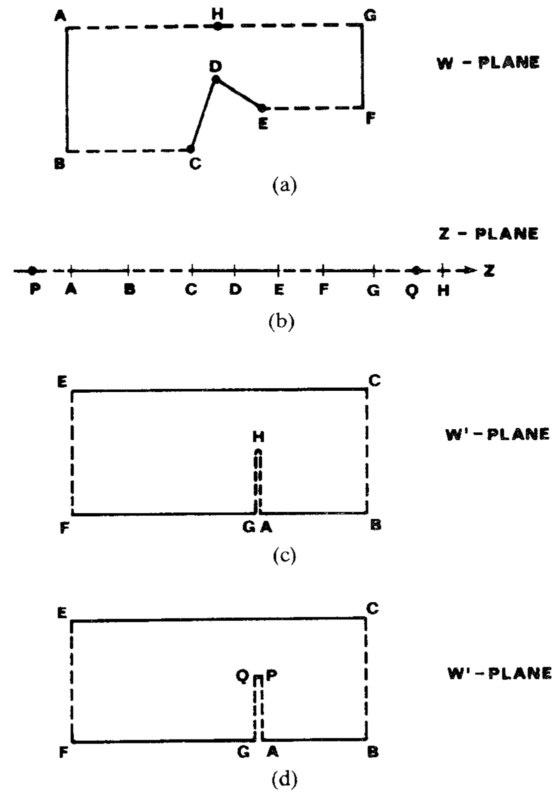

Fig. 5 Conformal transformation from a general three-wall geometry to a parallel-plate geometry. (a) Structure in the original w plane. (b) Intermediate z plane (c) Transformed geometry. (d) Geometry derived by an improved procedure.

Acknowledgment

The author wishes to thank the reviewers of the manuscript for their very useful suggestions.

References

[1] R Levy, "Conformal transformations combined with numerical techniques, with applications to coupled-bar problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp 369–375, Apr. 1980

[2] R Diaz, "The discrete variational conformal technique for the calculation of strip transmission-line parameters," *IEEE Trans. Microwave Theory Tech*, vol MTT-34, pp. 714–722, June 1986.

[3] E Costamagna, "On the numerical inversion of the Schwarz-Christoffel conformal transformation," *IEEE Trans. Microwave Theory Tech.*, vol MTT-35, pp. 35–40, Jan 1987

[4] E Costamagna, "TEM parameters of angular offset strip lines," *Alta Frequenza*, vol. LVII, no 5, pt I, pp. 193–201, June 1988.

[5] H. J Riblet, "Asymmetric even-mode fringing capacitance," *IEEE Trans Microwave Theory Tech*, vol MTT-25, pp. 410–412, May 1977

[6] D. L. Gish and O. Graham, "Characteristics impedance and phase velocity of a dielectric-supported air strip transmission line with side walls," *IEEE Trans. Microwave Theory Tech*, vol. MTT-18, pp. 131–156, Mar. 1970

[7] J. I. Smith, "The even- and odd-mode capacitance parameters for coupled lines in suspended substrate," *IEEE Trans. Microwave Theory Tech.*, vol MTT-19, pp 424–431, May 1971.

[8] E A Guillemin, *The Mathematics of Circuit Analysis* New York: Wiley, 1950, ch. VI, art. 25.

[9] H. J. Riblet, "The characteristic impedance of a family of rectangular coaxial structures with off-centered strip inner conductors," *IEEE Trans Microwave Theory Tech*, vol MTT-27, pp 294–298, Apr 1979.

[10] U Maltese, private communication, Marconi Italiana, 1971.